Triad Resonance in the Gravity–Acoustic Family

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Summary Resonance interactions of waves play a prominent role in energy share among the different wave types involved. Such interactions may significantly contribute, among others, to the evolution of the ocean energy spectrum by exchanging energy between surface-gravity waves; surface and internal gravity waves; or even surface and compression-type waves, that can transfer energy from the upper ocean through the whole water column reaching down to the seafloor.

Introduction

A resonant triad occurs among a triplet of waves, usually involving interaction of nonlinear terms of second order perturbed equations. It has been believed that in a homogeneous fluid a resonant triad is possible only when tension forces are included, or at the limit of a shallow water, and that when the compressibility of water is considered, no resonant triads can occur within the family of gravity–acoustic waves. However, recently it has been proved that resonant triads comprising two opposing surface-gravity waves of similar periods and a much longer acoustic–gravity wave, of almost double the frequency, exist [1-3].

Two gravity one acoustic triad

interaction differs fundamentally from a standard resonant triad, as the acoustic spatial scale is much longer than the gravity wavelength owing to the weak compressibility of water.

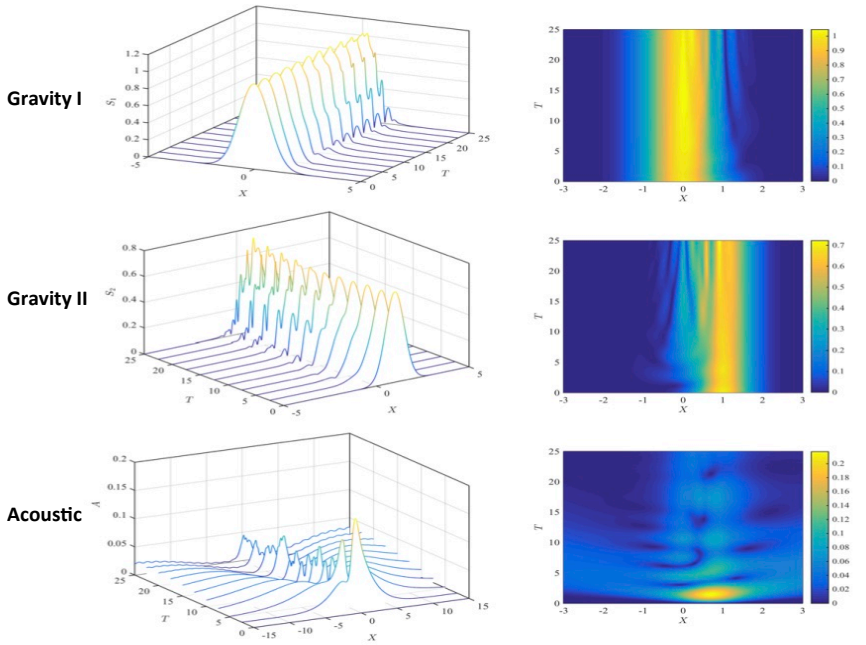

Figure 1 A qualitative example of spatio-temporal evolution of the wave triad amplitudes of two gravity wave packets interacting with an acoustic mode. The gravity waves (top and middle) generate an acoustic mode that displaces the energy in space. The envelopes of the gravity wave packets are practically standing while the acoustic envelope travels in both directions. As the interaction carries, the phases shift and the nonlinear interaction becomes inefficient.

A new class of triads

involving a gravity wave and two acoustic waves of almost double the length is found. Interestingly, the two acoustic waves propagate in the same direction with similar wavelengths, that are almost double of that of the gravity wave. The evolution of the wave triad amplitudes is periodic and it is derived analytically, in terms of *Jacobian elliptic* functions and elliptic integrals [4].

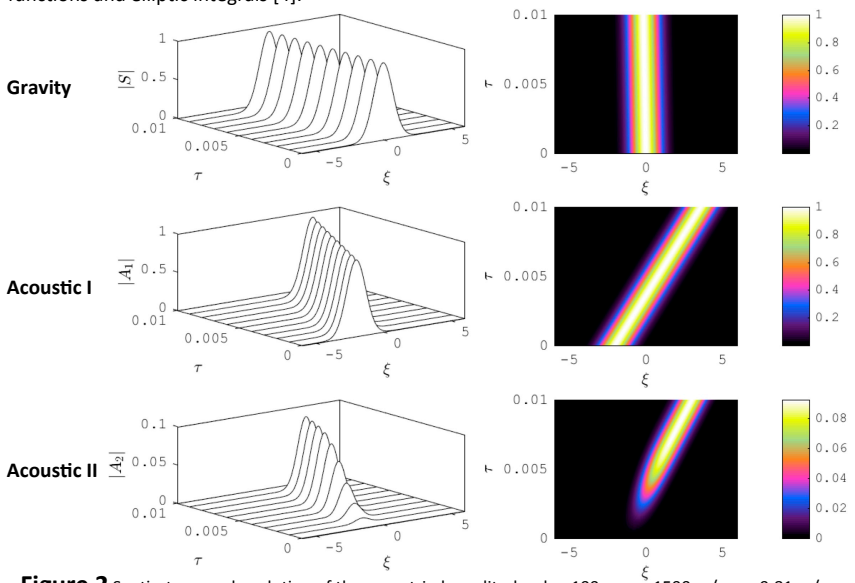

Figure 2 Spatio-temporal evolution of the wave triad amplitudes; h = 100 m, c = 1500 m/s, g = 9.81 m/s2, (σ, ω1, ω2) = (0.32, 25, 24.68) rad/s. The gravity wave (top) remains almost unaltered, while the envelope slowly displaces to the left. However, the prescribed acoustic envelope travels relatively fast to the right minimizing the interaction time. Consequently, the resultant acoustic wave envelope might be significantly smaller. As the two acoustic beams concurrently move away from the gravity wave, with disparate group velocities, the resonant interaction gradually vanishes.

Significance

The physical importance of this type of triad interactions is the modulation of pertinent acoustic signals, leading to inaccurate signal perceptions.

References

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