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A Probabilistic Approach for Predicting Average Slug Frequency in Horizontal Gas/Liquid Pipe Flow

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Résumé — Approche probabiliste de prévision de la fréquence moyenne des bouchons dans des écoulements gaz/liquide en conduite horizontale — Dans cet article nous présentons un modèle de prévision de la fréquence moyenne des bouchons dans des écoulements gaz/liquide en conduite horizontale. Le modèle prend en considération la probabilité de formation de bouchons si les bouchons sont déclenchés aux ventres d'une perturbation sinusoïdale, le long de la conduite, à la fréquence d'oscillation de l'interface. Il est estimé qu'un bouchon se forme, si et seulement si, il est déclenché à un espace-temps suffisamment éloigné des bouchons existants. Il est constaté que la probabilité de formation des bouchons diminue avec la distance à l'entrée, étant donné que le passage en aval des bouchons existants prévient la formation de nouveaux bouchons. Les prévisions obtenues par le modèle sont comparées aux mesures air/eau, fréon/eau et air/huile extraites de la littérature avec une concordance satisfaisante. Cependant, un écart par rapport aux mesures est observé lorsqu'un liquide de haute viscosité est considéré. Le modèle contribue à la prévision de régime d'écoulement des bouchons et peut servir de ligne directrice pour la conception d'écoulement gaz/liquide en conduite horizontale.

Abstract — A Probabilistic Approach for Predicting Average Slug Frequency in Horizontal Gas/liquid Pipe Flow *— In this paper, we present a model for predicting the average slug frequency in horizontal gas/liquid pipe flow. The model considers the probability of slug formation if slugs are triggered at the antinodes of a sinusoidal perturbation, along the pipe at the frequency of oscillation of the interface. A slug is assumed to form if and only if triggered at a space-time far enough from existing slugs. The probability of forming slugs is found to decrease with distance from the inlet, since the downstream passage of existing slugs prevents the formation of new slugs. Predictions by the model are compared with air/water, freon/water and air/oil measurements found in literature, with a satisfactory agreement. However, a deviation from measurements is observed when considering high viscosity liquid. The model contributes to the prediction of slug flow regime and can act as a guideline toward the design of gas/liquid horizontal pipe flow.*

INTRODUCTION

A slug flow pattern is commonly observed when transporting gas and liquid in horizontal and near horizontal pipe flows. Slug flow is characterized by plugs of liquid moving downstream separated by elongated bubbles moving along the top of the pipe.

The formation of slugs follows from surface waves evolving on the gas-liquid interface. One of the explanations of the occurrence of the surface waves is the presence of oscillations in the gas and liquid layers due to shear flow. Consequently, oscillations at the interface result in pressure and velocity fluctuations that drive the wave formation. At relatively large gas and liquid Froude numbers the pressure fluctuations cause a resonant forcing of the free surface modes, which then grow indefinitely [1]. If the flow rates are sufficiently high, the evolving waves can reach the top of the pipe and form slug flow.

Predictions of the Flow Conditions

Two main theories are involved for predicting the necessary flow conditions at which slugs may form: theories on the stability of stratified flow and the stability of slugs. [2] used the stability of stratified flow to describe waves on thin films over which air is blowing. Their analysis was followed by [3-7] who used the Kelvin-Helmholtz instabilities by analysing small sinusoidal perturbations on the interface of the stratified flow. This approach gives a criterion for the transition from stratified-smooth to wavy flow. On the other hand, the stability of slug flow concerns a volumetric balance between the liquid flow rate shedding from the back of a slug and the liquid rate accumulating at the front. This balance results in a minimum liquid area at the front that is required for a slug to be stable [8-10]. In this approach, the back of the slug is modelled as a bubble [11], which is supported by measurements and photographs by [12]. The two stability theories provide predictions of the flow conditions that are necessary for the formation of slug flow. However, they do not provide prediction of slug length or frequency. In this paper, we use the predictions by the two theories to obtain the properties of a stable slug flow.

Prediction of Slug Frequency

Slug frequency has been investigated by several researchers in the last decades. [13] modelled the slug frequency as a function of the superficial gas and liquid velocities and the liquid Froude number. The correlation by [13] is based on slug flow measurements performed in a wide range of pipe diameters, at relatively large liquid flow rates. Similar correlations based on additional data have been suggested by a number of researchers (*e.g.* [14-16]). [17] presented a semi–mechanistic model postulating that the slug frequency is one-half of the frequency of the unstable waves responsible for slug initiation. [18] reported that the postulation by [17] is inconsistent with their experimental data. [19] carried out measurements at very large liquid flow rates and suggested a slug frequency correlation directly proportional to the squared liquid Froude number. [16] presented a review of eight slug frequency prediction methods: seven correlations and the mechanistic model by [5]. The correlations were found unsatisfactory and the mechanistic model was computationally too demanding. [16] suggested a correlation which is basically the correlation by [13] extended to include positive pipe inclinations, relative to the horizontal. More recently, slug frequency has been examined by [20] who applied Poisson probability theory to model slug frequency in gas/liquid pipe flow; [21] who examined the chaotic behaviour of slug flow; [22] who revisited the slug frequency problem with the aim of providing a unified slug frequency relation valid for all available data; [23] who presented a new methodology for the control of slug frequency; [24] who investigated the effect of high viscosity oil on slug frequency; and [25] who investigated slug frequency in horizontal gas/liquid pipe flow.

In this paper, we present a model for predicting the average slug frequency in horizontal gas-liquid pipe flow using a probabilistic approach. For the validation of the model, we compare predictions of the slug frequency with measurements found in the literature [15, 26-28], for gas-liquid horizontal pipe flows.

Theoretical background including stability of stratified and slug flow is given in Section 1. The proposed slug frequency model is presented in Section 2. Section 3 provides comparisons between predictions by the slug frequency model and measurements. Finally, the conclusions are presented.

1 BACKGROUND

1.1 Stratified Flow Pattern

An idealized model of the stratified flow pattern is represented by a simplified geometry as given in Figure 1a. The diameter of the pipe is D. The height of the liquid layer along the centreline is h. The length of the segments of the pipe circumference in contact with the gas and liquid are S_G and S_L, respectively. The length of the gas-water interface is presented by S_i. The areas occupied by the gas and the liquid are A_G and A_L, respectively. Given the pipe diameter, these parameters are used to calculate the gas and liquid heights H and h of the fully developed stratified flow, by using geometric relations (*e.g.* [29]). The momentum balances for the gas and the liquid flows are:

$$-A_G\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right) - \tau_{WG}S_G - \tau_i S_i + \rho_G A_G g\sin\theta = 0 \quad (1)$$

and:

$$-A_L\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right) - \rho_L g\cos\theta\left(\frac{\mathrm{d}h}{\mathrm{d}x}\right) - \tau_{WL}S_L + \tau_i S_i + \rho_L A_L g\sin\theta = 0 \quad (2)$$

where ρ_G and ρ_L are the gas and the liquid densities, θ is the inclination angle of the pipe from the horizontal, $\mathrm{d}p/\mathrm{d}x$ is the pressure gradient, $\mathrm{d}h/\mathrm{d}x$ is the liquid hydraulic gradient, and g is the acceleration due to gravity. The time-averaged stress of the gas and liquid phases at the wall and the stress at the interface, τ_{WG}, τ_{WL} and τ_i, are defined in terms of friction factors:

$$\tau_{WG} = \frac{f_{WG}\rho_G U^2}{2};\ \tau_{WL} = \frac{f_{WL}\rho_L u^2}{2};\ \tau_i = \frac{f_i\rho_G (U-u)^2}{2} \quad (3)$$

where U and u are the average actual gas and liquid velocities, respectively. The wall gas and liquid friction factors,

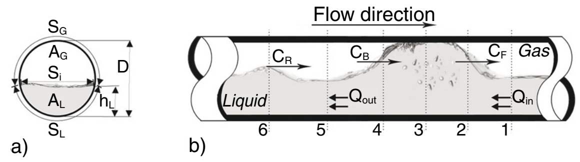

Figure 1

a) A simplified geometry of stratified flow. b) Properties of slug flow.

f_{WG} and f_{WL}, can be calculated from the Blasius equation for turbulent flow and a smooth pipe wall:

$$f_{WG} = 0.0791 Re_G^{-1/4}; \qquad f_{WL} = 0.0791 Re_L^{-1/4} \tag{4}$$

The interfacial friction factor, f_i, is calculated from the friction factor for a smooth surface, f_s. At relatively very low gas and liquid flow rates $f_i = f_s$. However, at larger flow rates, f_i becomes larger near the transition to slug flow, and it is estimated from the following relations [10, 30-32]:

$$\frac{f_i}{f_s} = 2, \quad \text{smooth surface} \quad (U-u) \leq (U-u)_{crit} \tag{5}$$

$$\frac{f_i}{f_s} = 5, \quad \text{wavy surface} \quad (U-u) \leq (U-u)_{crit} \tag{6}$$

$$\frac{f_i}{f_s} = 5+15\left(\frac{h}{D}\right)^{0.5}\left[\frac{(U-u)}{(U-u)_{crit}}-1\right], (U-u) > (U-u)_{crit} \tag{7}$$

The quantity $(U - u)_{crit}$ is the critical relative velocity at which waves become unstable, defined by:

$$(U-u)^2_{crit} = 2\frac{\rho_L}{\rho_G}\sqrt{\frac{\sigma g cos\theta}{\rho_L}} \tag{8}$$

The gas and liquid Reynolds number in Equation (4) are given by:

$$Re_G = \frac{D_{HG}U}{\nu_G}; \qquad Re_L = \frac{D_{HL}u}{\nu_L} \tag{9}$$

where ν_G and ν_L are the kinematic viscosities of gas and liquid, and D_{HG} and D_{HL} are the hydraulic diameters defined as:

$$D_{HG} = \frac{4A_G}{S_G+S_i}; \qquad D_{HL} = \frac{4A_L}{S_L} \tag{10}$$

1.2 Slug Stability Model

The slug stability model considers the rates of liquid adjoining or detaching from the slug at its front or rear. Slugs are stable (not decaying) when the rates of liquid adjoining are not less than the rates at which liquid detaches. Figure 1b gives an illustration of a slug moving with front velocity C_F over a stratified liquid layer at station 1 of area A_{L1} and actual velocity u_1. The volumetric flow rate of liquid adjoining the slug is:

$$Q_{in} = (C_F - u_1)A_{L1} \tag{11}$$

The rear of the slug is assumed to behave as a bubble moving with a velocity C_B, where:

$$C_B = U_{Mix} + 0.542\sqrt{gD}, \qquad U_{Mix} < 2\sqrt{gD} \tag{12}$$

$$C_B = 1.1U_{Mix} + 0.542\sqrt{gD}, \quad 2\sqrt{gD} < U_{Mix} < 3.5\sqrt{gD} \tag{13}$$

$$C_B = 1.2U_{Mix}, \qquad U_{Mix} > 3.5\sqrt{gD} \tag{14}$$

The volumetric flow rate of the liquid detaching from the slug is:

$$Q_{out} = (C_B - u_3)(1-\varepsilon)A, \qquad \text{at station 3} \tag{15}$$

The parameter u_3 is the actual liquid velocity at station 3, and ε is the volume fraction of the gas in the slug [12, 33]:

$$\varepsilon = 0.8\left(1 - \frac{1}{\left(1+(U_{Mix}/8.66)^{1.39}\right)}\right) \tag{16}$$

where U_{Mix} is the mixture velocity ($U_{Mix} = U_{SG} + U_{SL}$, where U_{SG} and U_{SL} are the superficial gas and liquid velocities, respectively). Assuming neutral stability, $Q_{in} = Q_{out}$ and $C_F = C_B$, and making use of Equations (11) and (15), the following relation is obtained:

$$\left(\frac{A_{L_1}}{A}\right)_{crit} = \frac{(C_B-u_3)(1-\varepsilon)}{(C_B-u_1)} \tag{17}$$

for the area of the stratified flow at the front. Using Equation (17) and geometric relations, the critical height, $h_{L_{crit}}$, at the slug front required for the slug to be neutrally stable is obtained. The detailed analysis of slug stability model is well documented by [10] and [34].

2 PROBABILISTIC SLUG MODEL

Oscillations in gas and liquid, due to shear flow, may result in perturbations oscillating on the interface of the stratified flow. Although the Kelvin-Helmholtz instability analysis of such perturbations can be applied for the prediction of the critical flow conditions for the formation of slug flow (*e.g.* [7]), the aim of the model presented here is to provide a probabilistic approach for the prediction of slug frequency, which cannot be governed by the Kelvin-Helmholtz approach. The current model postulates that slugs are triggered (though not necessarily form) at the frequency of oscillation $\mathbf{f}_{r,i}$, at the location of the antinodes of a sinusoidal perturbation $i = 1, ..., n$ along the pipe, as shown in Figure 2. The length of the pipe is L_{pipe}, and a characteristic distance between two neighbouring antinodes is the average length scale, $l_T = 0.07D_{HL}$ for a fully developed pipe flow. The

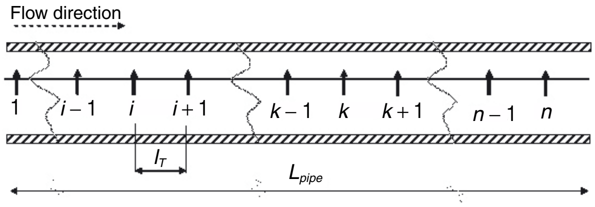

Figure 2

Sketch of triggering slug precursors along the pipe.

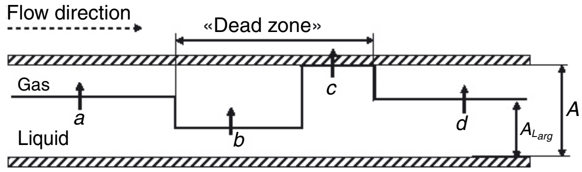

Figure 3

An example for triggering slug precursors at different locations *a*, *b*, *c* and *d*. The triggering at locations *a* and *d* may form slugs, whereas triggering at *b* and *c*, in the "dead zone", fails to form slugs.

probability of forming a slug is influenced by the passage of other slugs that have already been formed upstream in the pipe. Such slugs may prevent the triggering of new slugs. Therefore, it is required to calculate the probability of slug formation $\mathbb{P}$, given the passage of other slugs upstream in the pipeline.

2.1 Conditional Probability of Slug Formation

We assume that a slug will form if triggered at any location k unless another slug is passing at that location, at the triggering time, as illustrated in Figure 2. The passing slug unit (slug and bubble) is referred to as a "dead zone", where triggered slugs fail to form slugs. The initial length of the "dead zone" is defined as the length passed by a perturbation moving with the interface at the velocity C_R for a time period $\Delta t = 1/\mathbf{f}_{r,i}$:

$$l_D = C_R \Delta t \tag{18}$$

The wave velocity C_R is calculated from the classical Kelvin-Helmholtz analysis of a stratified flow (*e.g.* [32]):

$$C_R = \frac{\rho_G U h + \rho_L u H}{\rho_L H + \rho_G h} \tag{19}$$

where h and H are the heights of liquid and gas, respectively. Figure 3 shows examples of slug triggering that forms slugs (a and d), and others that fail to form slugs (b and c) due to their existence in the "dead zone" at the triggering time. We define a factor $m_{i,k}$ which is a measure for the effect of a slug at location i on the triggered slug precursor:

$$m_{i,k} = max\left[0, \frac{t_{i,w} - t_{i,F}}{\Delta t}\right] \tag{20}$$

where $t_{i,w}$ is the time that takes the upstream wave behind a slug at location i to reach the triggered slug precursor at location k:

$$t_{i,w} = \frac{(k-i)l_T}{C_R} \tag{21}$$

whereas $t_{i,F}$ is the time needed for the front of a slug at location i to reach the triggered slug precursor at location k:

$$t_{i,F} = \frac{(k-i)l_T - l_{dead}}{C_F} \tag{22}$$

Equation (20) provides the number of slugs that may form upstream and their passage will prevent the formation of a slug precursor that is triggered at location k, within the time interval $(t_{i,w} - t_{i,F})$. Therefore, the conditional probability of forming a slug (if triggered) at location k is:

$$\mathbb{P}_k = 1 - \sum_{i=1}^{k-1} \frac{m_{i,k}\mathbb{P}_i}{n} \tag{23}$$

where $\mathbb{P}_1 = 1$ is the conditional probability of forming a slug, if triggered, at location 1. Averaging the probability of slug formation along the pipe $\langle \mathbb{P}_k \rangle$, the slug frequency is obtained as follows:

$$\mathbf{f}_S = \langle \mathbb{P}_k \rangle \mathbf{f}_{r,i}, \qquad \text{for all } k = 1, ..., n \tag{24}$$

2.2 Frequency of Oscillation

It is known that the first harmonic dominates the resonance frequencies, since the wave damping factor grows proportionally to the square root of the frequency [35]. Thus, for the frequency of oscillations in gas, liquid and interface, we write:

$$\mathbf{f}_{r,G} = \frac{u_{\tau,G}}{H}, \quad \mathbf{f}_{r,L} = \frac{u_{\tau,L}}{h}, \quad \mathbf{f}_{r,i} = u_{\tau,i}\left(\frac{1}{H} + \frac{1}{h}\right) \tag{25}$$

where the friction velocity in gas, liquid and at the interface is respectively given by:

$$u_{\tau,G} = \sqrt{\frac{\tau_{WG}}{\rho_G}}, \quad u_{\tau,L} = \sqrt{\frac{\tau_{WL}}{\rho_L}}, \quad u_{\tau,i} = \sqrt{\frac{\tau_i}{\rho_L}} \tag{26}$$

Substituting the interfacial terms in Equations (26) and (25) into Equation (24) finally yields:

$$\mathbf{f}_S = \langle \mathbb{P}_k \rangle \sqrt{\frac{\tau_i}{\rho_L}}\left(\frac{1}{H} + \frac{1}{h}\right) \tag{27}$$

Substituting τ_i into Equation (26) and expressing the result as function of $\mathbf{f}_{r,G}$ and $\mathbf{f}_{r,L}$, we obtain a relation for an interfacial frequency as function of oscillations in gas and liquid:

$$\frac{\mathbf{f}_{r,i}}{\sqrt{f_i}} = \sqrt{\frac{\rho_G}{\rho_L}}\left(\frac{\mathbf{f}_{r,G}}{\sqrt{f_{WG}}}H - \frac{\mathbf{f}_{r,L}}{\sqrt{f_{WL}}}h\right)\left(\frac{1}{H} + \frac{1}{h}\right) \tag{28}$$

Note that an expression for the average fully developed length of a slug unit L_U (bubble and slug) can be obtained by dividing the bubble velocity, C_B, by the slug frequency (*Eq. 27*) (*i.e.*, $L_U = C_B/\mathbf{f}_S$).

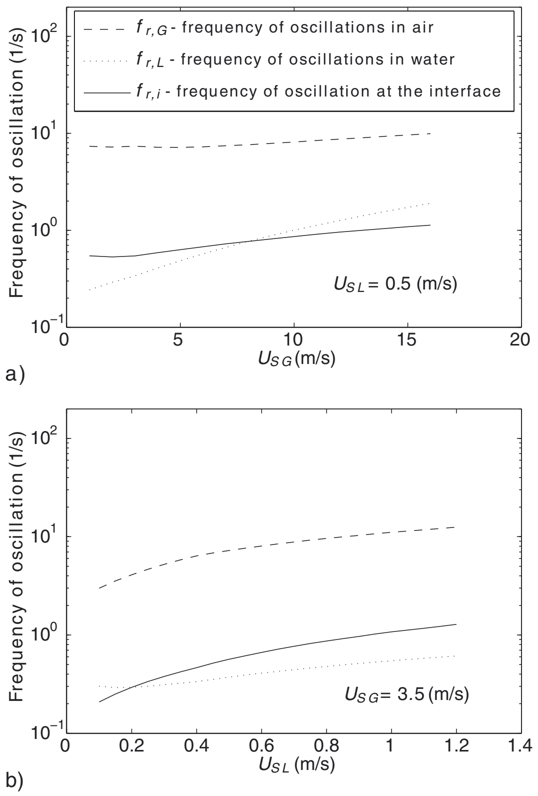

Figure 4

Theoretical calculations of the frequency of oscillations in air, water and at the interface. D = 0.095 m, L_{pipe} = 24 m. a) Constant liquid flow rate, U_{SL} = 0.5 m/s. b) Constant gas flow rate, U_{SG} = 3.5 m/s.

3 RESULTS

Theoretical, calculations of the frequency of oscillations in the gas and liquid phases, and at the interface are given in Figure 4. The calculations were performed for air-water flow in a 24 m long pipe with 0.095 m i.d. The subplots a) and b) indicate constant U_{SL} = 0.5 m/s and U_{SG} = 3.5 m/s, respectively. The dashed and dotted curves are calculations of the frequency of the oscillations generated in the gas and liquid, respectively; whereas the solid curve is the calculation of the frequency at the interface. In Figure 4, the frequency of the oscillations generated in the gas phase, $\mathbf{f}_{r,G}$, is order of magnitude larger than that in the liquid, $\mathbf{f}_{r,L}$, or at the interface, $\mathbf{f}_{r,i}$. However, the relative behaviour of $\mathbf{f}_{r,i}$ with the flow rates is similar to that of $\mathbf{f}_{r,G}$. This might be an indication that the momentum of the liquid phase dictates the magnitude of the interfacial oscillations, whereas the intensity of oscillations in the gas phase dictates the behaviour of $\mathbf{f}_{r,i}$ with the flow rates.

Figure 5 presents theoretical calculations with the above model of the conditional probability of slug formation as function of the location on the pipe (detailed steps are given in Appendix). The pipe is horizontal, 137 m long with a diameter of 0.052 m, and the fluids are air and water with U_{SG} = 3.5 m/s. The probability of slug formation corresponds to Equation (23). In Figure 5, the probability monotonically decreases downstream in the pipe, which is due to the more probable passage of slugs formed upstream (see *Eq. 23*). It is well known that increasing U_{SL} results in the formation of slugs upstream at higher frequency (*e.g.*, [18]). The passage of the increased number of slugs prevents the formation of a larger number of slugs downstream. Thus, while the average slug frequency increases with U_{SL} the conditional probability of slug formation, at any downstream location, decreases.

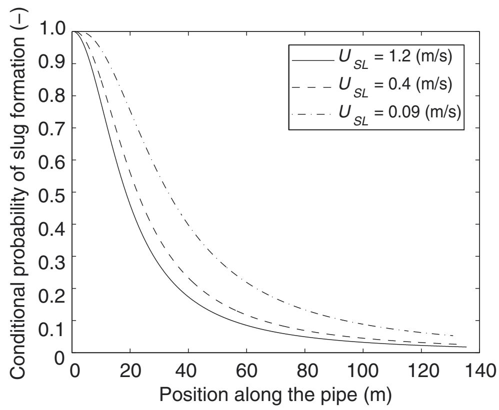

Figure 5

Theoretical predictions of the conditional probability of slug formation in air-water horizontal pipe flow. U_{SG} = 3.5 m/s, D = 0.052 m, L_{pipe} = 137 m.

Theoretical predictions of slug frequency are compared with measurements in Figure 6. The measurements were carried out by [26] and [27] (published in [36]) at U_{SL} = 1.2, 0.8 and 0.5 m/s, and a range of $1 < U_{SG} < 16$ m/s. The pipe is 24 m long with a diameter D = 0.095 m. The agreement between predictions and measurements is satisfactory. Equation (24) successfully predicts that the slug frequency increases with increasing U_{SG}. However, a systematic overprediction is noticed as U_{SL} decreases. It is also noticeable that the region $U_{SG} > 10$ m/s is a transition region to annular flow. In order to improve the accuracy of the predictions of slug frequency, the effect of annular flow needs to be considered.

Figure 7 compares theoretical predictions with air/water measurements in pipes with diameters of 0.042 m [15] and 0.095 m [26, 27] and with Freon/water measurements in a pipe diameter of 0.15 m [28] (see *Tab. 1*). The ordinate is $f_s D/U_{SL}$ and the abscissa is U_{SL}/U_{Mix}. The current model successfully predicts the slug frequency for the different pipe diameters.

Figure 8 compares theoretical predictions of the current model (solid curves) with predictions by [13] (dashed curves) and measurements by [24], for air/oil pipe flow.

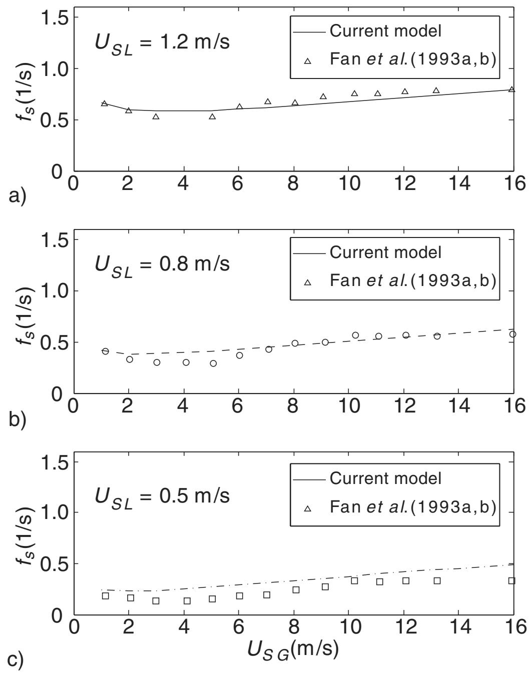

Figure 6

Theoretical predictions and measurements of slug frequency at fixed U_{SL} and varying U_{SG}. D = 0.095 m, L_{pipe} = 24 m. The measurements were performed by [26] and [27]. a) U_{SL} = 1.2 m/s, b) U_{SL} = 0.8 m/s, c) U_{SL} = 0.5 m/s.

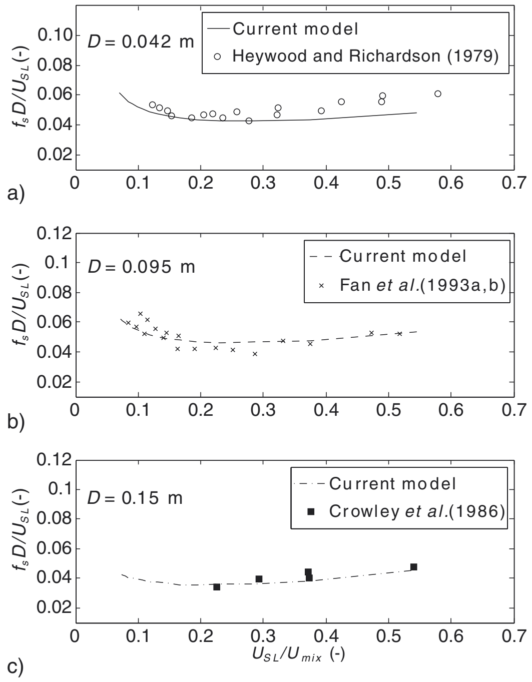

Figure 7

Theoretical predictions and measurements of slug frequency for different pipe diameters. a) D = 0.042 m, b) D = 0.095 m, c) D = 0.15 m.

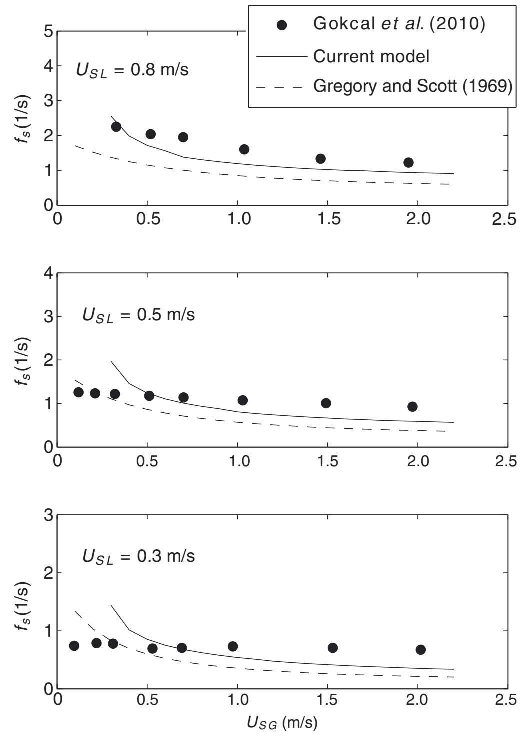

Figure 8

Theoretical predictions and measurements of slug frequency of high viscosity oil at fixed U_{SL} and varying U_{SG}, D = 0.0508 m, L_{pipe} = 18.9 m, μ = 0.181 Pa·s.

The pipe diameter and length are D = 0.0508 m and L_{pipe} = 18.9 m, respectively. The liquid viscosity is 0.181 Pa·s. For $U_{SG} > 0.5$ m/s, the current model provides an improved prediction of slug frequency, compared to the correlation by [13]. However, a systematic deviation from measurements is noticed. A possible reason for this deviation is the relatively high viscosity of the liquid, which significantly affects parameter such as length scale (l_T) and bubble velocity (C_B).

A summary of the slug frequency data set used for comparison with predictions is given in Table 1. The parameters ϵ_{avg} and ϵ_{max} are the average and maximum errors in predictions, calculated for all the measurements presented in Figure 8. The maximum deviation from measurements by [24] reach as high as 43%. Further increase in the liquid viscosity is expected to increase the deviation, as discussed earlier.

CONCLUSIONS

1. A probabilistic based model for predicting the average slug frequency in gas-liquid horizontal pipe flow was introduced. The model considers the probability of slug formation if slugs are triggered at the antinodes of a sinusoidal perturbation, along the pipe, at the frequency of oscillation of the perturbed interface. Predictions by

TABLE 1

Summary of slug frequency data sets

Source	U_{SL} (m/s)	U_{SG} (m/s)	D (m)	L_{pipe} (m)	Gas/liquid	ϵ_{avg}	ϵ_{max}
[28]	0.2-1.4	1-16	0.15	20	Freon/water	−0.03	−0.11
[26]	0.5-1.2	1-16	0.095	24	Air/water	0.01	0.21
[24]	0.3-0.8	0.1-2	0.0508	18.9	Air/oil	−0.09	0.43
[15]	0.2-1.4	0.1-8	0.042	14	Air/water	−0.10	−0.23

the model were compared with slug frequency measurements, found in literature, with a satisfactory agreement.

2. The model was validated by comparing slug frequency calculations with measurements found in literature. The agreement between predictions and measurements is satisfactory at $U_{SG} < 10$ m/s. However, at large gas or low liquid flow rates the model overpredicts the slug frequency. A possible reason for this deviation between predictions and measurements is the transition to annular flow, gas entrainment and liquid breakup that is not considered in the paper.
3. The probability of forming a slug decreases in the downstream part of the pipe as a result of the passage of slugs formed upstream. It also decreases along the pipe when the liquid velocity increases, which is due to increasing the length of the “dead zones”.
4. At larger pipe diameters the minimum length of a stable slug increases. This results in larger “dead zones”, at which triggering fails to form a slug. Hence, the probability of forming a slug decreases with increasing the pipe size.
5. The oscillations at the interface $\mathbf{f}_{r,i}$ have a logarithmic behaviour (with the flow rates) similar to the frequency of oscillations in the gas phase, $\mathbf{f}_{r,G}$ (see *Fig. 4*). However, the order of magnitude of $\mathbf{f}_{r,i}$ is comparable with the frequency of oscillations in the liquid, $\mathbf{f}_{r,L}$. A possible explanation for this behaviour is that the magnitude of the interfacial oscillations is imposed by the momentum of the liquid phase as the interfacial friction velocity (*Eq. 26*) is dominated by the liquid ($u_{\tau,i} = \sqrt{\tau_i/\rho_L}$ since $\rho_L \gg \rho_G$).
6. Note that the presented model does not consider the geometry of the inlet which in many cases has direct influence on the generation of slugs, in particular the short hydrodynamic slugs. However, for relatively smooth gas and liquid flow entrance (*e.g.* at low gas and liquid flow rates), where the effect of gas jet on the interface is secondary [37], slugs develop from smooth stratified flow further downstream in the pipe [18]. In this respect, the current model is applicable for both hydrodynamic and terrain slugging.
7. Finally, the current model can be used for stochastic analysis of predicted values of slug frequency. However, in this case, a modification of the factor $m_{i,k}$ is required:

$$m_{i,k} = max\left[0, \frac{min\left[t_k, t_{i,w}\right] - t_{i,F}}{\Delta t}\right] \tag{29}$$

where t_k is the instant at which a slug precursor is triggered at location k. Here, t_k is a random array containing all triggering instants at location k.

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APPENDIX A STEPS FOR CALCULATING THE CONDITIONAL PROBABILITY OF SLUG FORMATION, $\mathbb{P}_K$

Provided the fluids properties, gas and liquid superficial velocities and pipe length and diameter, the following steps are carried out in order to calculate $\mathbb{P}_k$:

a) the average gas and liquid heights H and h of the fully developed stratified flow are obtained using the momentum balances Equations (1) and (2), and the geometric relations by [29];
b) substitute the superficial gas and liquid velocities and pipe diameter in Equations (12–14) to find the bubble velocity C_B ($C_F = C_B$);
c) calculate the wave velocity C_R using Equation (19);
d) calculate the frequency of oscillation at the interface $\mathbf{f}_{r,i}$ from Equation (25) and substitute in $\Delta t = 1/\mathbf{f}_{r,i}$;
e) substitute the values of Δt and C_R in Equation (18) to obtain the length of the "dead zone", l_{dead}; and the liquid hydraulic diameter to obtain the average length scale ($l_T = 0.07D_{HL}$);
f) calculate $t_{i,w}$ and $t_{i,F}$ from Equations (21) and (22), respectively;
g) find the factor $m_{i,k}$ for each location $i < k$ using Equation (20) and substitute in Equation (23) to obtain the conditional probability of slug formation at location k;
h) repeat steps f) and g) for each $1 < k \leq n$.